Modeling Semantic Competence: A Critical Review of Frege's Puzzle about Identity

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Abstract. The present paper discusses Frege's Puzzle about Identity as an argument against a Millian theory of meaning for proper names. The key notion here is semantic competence. Strict notions of semantic competence are extrapolated from a two-sorted first-order epistemic logical modeling of a cognitive neuropsychological theory of the structure of lexical competence. The model allows for a rigorous analysis of Frege's argument. The theory and model of lexical semantic competence includes a multitude of types of competence, each yielding a different argument, far from all being as decisive against Millianism as has been the mainstream assumption in 20th century philosophy of language.

1 Introduction

In his 1892 paper *On Sense and Reference*, Gottlob Frege developed an argument against so-called *Millian* theories of meaning for proper names, later denoted *Frege's Puzzles about Identity*, cf. [11]. The Millian theory derives from philosopher John Stuart Mill, and equates meaning with reference. The view states that the meaning of a given, unambiguous proper name is constituted solely by the object to which the name refers, i.e. by its referent. On this view, the meaning of the name 'Hesperus' (the Evening Star) is constituted by the planet Venus construed as an existing object, and nothing more. Against this view, Frege's Puzzle can be formulated as follows. Consider the two true identity statements

(a) Hesperus is Hesperus
(b) Hesperus is Phosphorus

Given that the two names co-refer (Phosphorus, the Morning Star, does in fact denote the planet Venus), the two identity statements have the same meaning and must therefore be equally informative to a semantically competent speaker of English. As the first is a trivial validity of self-identity, this does not carry informational content. Opposed to this, the latter is a contingent, empirical fact, and may hence convey information. Hence, (a) and (b) *do* differ in informational content, and the Millian view should be rejected. The argument can be pinned out as follows:

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(A) (a) and (b) mean the same.

(A→B) If (a) and (b) mean the same, then a semantically competent speaker would know that (a) and (b) mean the same.

(B→C) If a semantically competent speaker would know that (a) and (b) mean the same, then they are equally informative to the speaker.

(¬C) (a) and (b) differ in informativeness to the competent speaker.

∴ Contradiction.

The four premises are jointly inconsistent, and the typical textbook choice[1] is to reject the premise (A). This premise is a consequence of the Millian view, and the conclusion drawn is that there must be more to meaning than mere reference.

In this paper, the above argument will be given a critical evaluation focusing on the notion of *semantic competence*, and doing so in an epistemic light. This is done by constructing a formal theory which includes the fundamental elements of the argument: an agent which has knowledge of the *objects* of the world it inhabits as well as a *basic language* for the agent. Further, the agent's knowledge of the meaning of the terms from its language is explicitly modeled. These things are included in order to gain the expressibility required to express the premises of the argument above wholly within the syntax of the formal language constructed.

The paper is organized as follows: first, a theory of semantic competence based on empirical evidence from cognitive neuropsychology is presented. This theory is then modeled using two-sorted first-order epistemic logic, a formal counterpart to Millian meaning is introduced, and strict notions of semantic competence are identified. In the ensuing section, the above argument will be evaluated using the identified notions of competence. Due to limitations of space, proof theory will not be considered. A complete axiomatization can easily be constructed based on the general completeness result for many-sorted modal logics from [15].

2 Lexical, Semantic Competence

Semantic competence is not in general a well-defined term, and its usage far from normalized. In the present, the notion is used as an objective measure, which allows for comparison of agents with respect to their individual competence. This is in contrast with the view of semantic competence used in e.g. [14], which depends on both subjective status and social context. It should further be noted that it is assumed that a satisfactory notion of semantic competence *simpliciter* cannot be found. Rather, it is assumed that agents will be semantically competent *with respect to* some part of language – be it a language, a sentence or a set of sentences or lexical items.

[1] See, for example, [2] or [10].

2.1 The Structure of Semantic Competence

The notion(s) of semantic competence invoked here are adopted from [12]. There, Diego Marconi constructs a conceptual theory of the *s*tructure of semantic, *l*exical *c*ompetence (SLC). The focal point is *lexical* competence, understood as competence with respect to words, as opposed to e.g. a truth-theoretic account of semantic competence, where competence consists in knowledge of T-sentence. See [9, ch. 9], [15] and [16] for critique of such theories. In structure, the theory is close to that of [3], but is deemed stronger as it is more precise and has empirical backing from studies in cognitive neuropsychology.[2] Time has not permitted a proper survey of literature from cognitive neuropsychology and related fields, and it is therefore unknown if this theory is inconsistent with newer findings or has been surpassed by later developments.

The elements of the theory consist of three relations defined over four ontologies. Each of the three relations correspond to a competence type. These are *inferential competence* and two types of *referential competence*, being *naming* and *application*. The four ontologies include one of *external objects*, one of *external words* (e.g., spoken or written words) and two mental modules: a *word lexicon* and a *semantic lexicon*. This structure is illustrated in Figure 1.

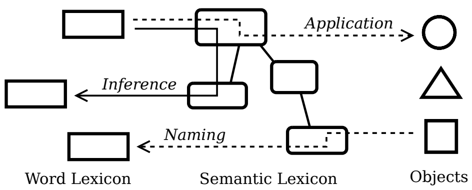

Fig. 1. A simplified illustration of the SLC. Elements in the word lexicon are not connected, only elements in the semantic lexicon are. External words are not pictured.

Ex.: Inferential competence requires connecting two items from the word lexicon *through* the semantic lexicon.

Word Lexicon, Semantic Lexicon and Inferential Competence. *Inferential competence* is the ability to correctly connect lexical items via the semantic lexicon, "underlying such performances as semantic inference, paraphrase, definition, retrieval of a word from its definition, finding a synonym, and so forth" [12, p. 59]. As such, inferential competence with respect to a given word consists "in the knowledge of true sentences in which [the] word is used" [p.58]. Hence, inferential competence is not a matter of logical proficiency and deductive skill, but rather depends on how well-connected the mental structure of the agent is. If the mental structure of the agent connects a given word in a way suitable for the agent to perform tasks like those mentioned, the agent is deemed inferentially competent with respect to that word.

To illustrate the competence form and introduce the modules required, assume an agent was to perform one of the mentioned tasks, namely finding a synonym for some name. On input, the external word (e.g., a name written on a piece of paper) is first analyzed and related to an mental representation from the word

[2] For the review of these studies, arguments for the structure and references to relevant literature, the reader is referred to [12].

lexicon. In [12], two word lexica are included for different input, a phonetic and a graphical. Here, attention is restricted to a simplified structure with only one arbitrary such, as illustrated in Figure 1, consisting only of proper names.[3] Using a graphical lexicon as an example, the word lexicon consists of the words an agent is able to recognize in writing. In the second step towards finding a synonym, the mental representation of the word is related to a mental concept in the semantic lexicon (or semantic system, in the terminology of [5]). The semantic lexicon is a collection of non-linguistic, mental concepts possessed by the agent, *distinct* from the word lexicon. The semantic lexicon reflects the agent's mental model of the world, and the items in this lexicon stand in various relations to one another. In contrast, in the word lexicon connections between the various items *do not exist*. Such only exist *via* the semantic lexicon. The third step is exactly a connection between two entries in the semantic lexicon. As the agent is to produce a synonym, this connection is assumed to be the reflexive loop. Finally, the reached note is connected to an entry in the word lexicon distinct from the input name, and output can be performed.[4]

Referential Competence and External Objects. *Referential competence* is "the ability to map lexical items onto the world" [12, p.60]. This is an ability involving all four ontologies, the last being *external objects*. It consists of two distinct subsystems. The first is *naming*. This is the act of retrieving a lexical item from the word lexicon when presented with an object. Naming is a two-step process, where first the external object is connected to a suitable concept in the semantic lexicon, which is then connected to a word lexicon item for output. The ability to name is required for correctly answering questions such as 'what is this called?'

The second subsystem is that of *application*. Application is the act of identifying an object when presented with a word. Again, this is a two-stage process, where first the word lexicon item is connected to a semantic lexicon item, which is then connected to an external object. The ability to apply words is required for correctly carrying out instructions such as 'hand me the orange.'

A naming or application deficit can occur if either stage is affected: if, e.g., either an object is not mapped to a suitable concept due to lack of recognition, or a suitable concept is not mapped to the correct (or any) word, then a naming procedure will not be successfully completed.

Empirical Reasons for Multiple Lexica and Competence Types. Marconi's structure of lexical competence may seem overly complex. It may be questioned, for example, why one should distinguish between word and semantic type modules, or why referential competence is composed of two separate competence types, instead of one bi-directional. These distinctions are made, however, as

[3] To only include proper names is technically motivated, as the modeling would otherwise require second-order expressivity. This is returned to below.

[4] For simplicity, a distinction between input and output lexica will not be made. See [17] for discussion.

empirical studies from cognitive neuropsychology indicate that the separation of these systems is mentally real, cf. [12, ch. 3]. In these studies, reviews of subjects with various brain-injuries indicate that these modules and abilities of human cognition are separate, in the sense that an ability may be lost or acutely impaired as a result of brain damage, while the other is left largely unaffected.

The distinction between word lexicon and semantic lexicon is also made in [13,17], and is supported in [5] by cases where patients are able to recognize various objects, but are unable to name them (they cannot access the word lexicon from the semantic lexicon). In the opposite direction, cases are reported where patients are able to reason about objects and their relations when shown objects, yet unable to do the same when prompted by their names (i.e., the patients cannot access the semantic lexicon from the word lexicon). The latter indicates that reasoning is done with elements from the semantic lexicon, rather than with items from the word lexicon.

Regarding competence types, it is stressed in [12] that inferential and referential competence are distinct abilities. Specifically, it is argued that the ability to name an object does not imply inferential competence with the used name, and, *vice versa*, that inferential knowledge about a name does not imply the ability to use it in naming tasks. No conclusions are drawn with respect to the relationship between inferential competence and application. Further, application is dissociated from naming, in the sense that application can be preserved while naming is lost. No evidence is presented for the opposite dissociation, i.e. that application can be lost, but naming maintained.

The model constructed in the ensuing section respects these dissociations. Space does not permit a long validation of the constructed model, but for this purpose, the reader can refer to [15].

3 Modeling the Structure of Lexical Competence

To model the structure from the previous section, a two-sorted first-order epistemic logic will be used. Do to limitations of space, only the absolutely required elements for the analysis of the argument from the introduction are included, though the syntax and semantics could easily be extended to include more agents, sorts, function- and relation symbols, cf. [15].

A two-sorted language is used to ensure that the model respects the dissociation of word lexicon and semantic lexicon. The first sort, σ_{OBJ}, is used to represent external objects and the semantic lexicon entries. As such, these are *non-linguistic* in nature. The second sort, σ_{LEX}, is used to represent the lexical items from the agent's language and entries in the word lexicon. Had terms been used to represent both simultaneously, the model would be in contradiction with empirical evidence.

The choice of quantified epistemic logic fits well with the Marconi's theory, if one assumes the competence types to be (perhaps implicitly) knowledge-based. The notions of object identification required for application is well-understood as modeled in the quantified $\mathsf{S5}$ framework, cf. [4]. The 'knowing who/what' *de re* constructions using quantified epistemic logic from [7] captures nicely the knowledge required for object identification by the subjects reviewed in [12]. These constructions will be returned to below.

3.1 Syntax

Define a language $\mathcal{L}$ with two sorts, σ_{OBJ} and σ_{LEX}. For sort σ_{OBJ}, include

1. $OBJ = \{a, b, c, ...\}$, a countable set of *object constant symbols*
2. $VAR = \{x_1, x_2, ...\}$, a countably infinite set of *object variables*

The set of terms of sort σ_{OBJ} is $TER_{OBJ} = OBJ \cup VAR$. For sort σ_{LEX}, include

1. $LEX = \{n_1, n_2, ...\}$, a countable set of *name constant symbols*
2. $VAR_{LEX} = \{\dot{x}_1, \dot{x}_2, ...\}$, a countably infinite set of *name variables*

The set of terms of sort σ_{LEX} is $TER_{LEX} = LEX \cup VAR_{LEX}$. Include further in $\mathcal{L}$ a unary function symbol, μ, of sort $TER_{LEX} \longrightarrow TER_{OBJ}$. The set of all *terms, TER,* of $\mathcal{L}$ are $OBJ \cup VAR \cup LEX \cup VAR_{LEX} \cup \{\mu(t)\}$, for all $t \in LEX \cup VAR_{LEX}$. Finally, include the binary relations symbol for identity, $=$. The well-formed formulas of $\mathcal{L}$ are given by

$$\varphi ::= (t_1 = t_2) \,|\, \neg\varphi \,|\, \varphi \wedge \psi \,|\, \forall x\varphi \,|\, K_i\varphi$$

The definitions of the remaining boolean connectives, the dual operator of K_i, $\hat{K}_i$, the existential quantifier and free/bound variables and sentences are all defined as usual. Though a mono-agent system, the operators are indexed by i to allow third-person reference to agent i.

3.2 Semantics

Define a model to be a quadruple $M = \langle W, \sim, Dom, \mathcal{I}\rangle$ where

1. $W = \{w, w_1, w_2, ...\}$ is a set of *epistemic alternatives* to actual world w.
2. $\sim$ is an *indistinguishability (equivalence) relation* on $W \times W$.
3. $Dom = Obj \cup Nam$ is the *(constant) domain of quantification*, where $Obj = \{d_1, d_2, ...\}$ is a non-empty set of *objects*, and $Nam = \{\dot{n}_1, \dot{n}_2, ..., \dot{n}_k\}$ is a finite, non-empty set of *names*.[5]
4. $\mathcal{I}$ is an *interpretation function* such that

$$\mathcal{I} : OBJ \times W \longrightarrow Obj \,|\, \mathcal{I} : LEX \longrightarrow Nam \,|\, \mathcal{I} : \{\mu\} \times W \longrightarrow Obj^{Nam}$$

Define a *valuation function*, v, by

$$v : VAR \longrightarrow Obj \,|\, v : VAR_{LEX} \longrightarrow Nam$$

[5] The set of names is assumed finite to be, in principle, learnable for a finite agent.

and a *x-variant of* v as a valuation v' such that $v'(y) = v(y)$ for all $y \in VAR_{(LEX)}/\{x\}$.

Based on the such models, define the truth conditions for formulas of $\mathcal{L}$ as follows:

$$\begin{array}{lll} M, w \models_v (t_1 = t_2) & \text{iff} & d_1 = d_2 \\ & & \text{where } d_i = \begin{cases} v(t_i) & \text{if } t_i \in VAR \cup VAR_{LEX} \\ \mathcal{I}(w, t_i) & \text{if } t_i \in OBJ \\ \mathcal{I}(t_i) & \text{if } t_i \in LEX \end{cases} \\ M, w \models_v \varphi \wedge \psi & \text{iff} & M, w \models_v \varphi \text{ and } M, w \models_v \psi \\ M, w \models_v \neg\varphi & \text{iff} & \text{not } M, w \models_v \varphi \\ M, w \models_v K_i\varphi & \text{iff} & \text{for all } w' \text{ such that } w \sim w',\ M, w' \models_v \varphi \\ M, w \models_v \forall x \varphi(x) & \text{iff} & \text{for all } x\text{-variants } v' \text{ of } v,\ M, w \models_{v'} \varphi(x) \end{array}$$

Comments on the semantics are postponed to the ensuing section.

Logic. A sound and complete two-sorted logic for the presented semantics can be found in [15]. The logic is here denoted $\mathsf{QS5}_{(\sigma_{LEX}, \sigma_{OBJ})}$. As of now, no arguments have been provided to the effect that $\mathsf{QS5}_{(\sigma_{LEX}, \sigma_{OBJ})}$ reflects the SLC or it's properties. This is focus of the ensuing section.

4 Correlation between Conceptual Theory and Formal Model

We argue that $\mathsf{QS5}_{(\sigma_{LEX}, \sigma_{OBJ})}$ represent the structure and properties of the SLC in two steps. First, it is shown by model-theoretic considerations that the logic, albeit indirectly, represent the ontologies of the SLC. Secondly, it is shown that the model can express the three competence types and that the dissociation properties are preserved in the logic. Before moving on to the latter, the interpretation of the function symbol μ as a *Millian meaning function* is presented.

4.1 Ontologies

The two sets of external objects and external words are easy to identify in the semantic structure. The external objects constitute the sub-domain *Obj*, and are denoted in the syntax by the terms TER_{OBJ}, when these occur outside the scope of an operator. External words (proper names) constitute the sub-domain Nam denoted by the terms TER_{LEX}, when occurring outside the scope of an operator.

The word lexicon and the semantic lexicon are harder to identify. The strategy is to extract a suitable notion from the already defined semantic structure. These constructs will not be utilized explicitly later on, but are included in order to validate the correctness of the modeling. To bite the bullet, we commence with the more complicated semantic lexicon.

Semantic Lexicon. No corresponding notion to the semantic lexicon have been defined so far, but it may be extrapolated from the introduced formalism. In order to include a befitting notion, define an *object indistinguishability relation* $\sim_w^a$:

$$d \sim_w^a d' \text{ iff } \exists w' \sim w : \mathcal{I}(a, w) = d \text{ and } \mathcal{I}(a, w') = d'.$$

and from this define the agent's *individual concept class for a at w* by

$$C_w^a(d) = \{d' : d \sim_w^a d'\}.$$

The semantic lexicon of agent i may then be defined as the collection of non-empty concept classes: $\mathsf{SL}_i = \{C_w^a(d) : C_w^a(d) \neq \emptyset\}$.

The set $C_w^a(d)$ consists of the objects indistinguishable to the agent by a from object d in the part of the given model connected to w by $\sim$. As an example, consider a scenario with two cups (d and d' from Obj) upside down on a table, where one cup conceals a ball. Let a denote the cup containing the ball, say d, so $\mathcal{I}(a, w) = d$. If the agent is not informed as to which of the two cups contain the ball, there will be an alternative w' to w such that $\mathcal{I}(a, w') = d'$. Hence, $d \sim_w^a d'$ so $d' \in C_w^a(d)$. The interpretation is that the agent cannot tell cups d and d' apart *with respect to which conceals the ball.*[6]

It is worth noting the object indistinguishability relation is *not* an equivalence relation, though this would fit nicely with the $\mathsf{S5}$ interpretation of knowledge. The lack of equivalence can be seen from the fact that $\sim_w^a$ is not guaranteed to be reflexive as possibly $\mathcal{I}(a, w) \neq d$ for all w.

Properties of such defined individual concepts can be expressed in $\mathcal{L}$. In particular, it is the case that

$$M, w \models_v \hat{K}_i(a = b) \text{ iff } \mathcal{I}(b, w') \in C_w^a(\mathcal{I}(a, w)),$$

i.e. agent i finds it possible that a and b are the same object iff b belongs to i's individual concept for a. Further,

$$M, w \models_v K_i(a = b) \text{ implies } C_w^a(\mathcal{I}(a, w)) = C_w^b(\mathcal{I}(b, w)),$$

i.e. if agent i knows two objects to be the same, then the their individual concept classes are identical. Finally, it is guaranteed that

$$|C_w^a(d)| = 1 \text{ iff } M, w \models_v \exists x K_i(x = a), \tag{1}$$

i.e. the agents has a singleton concept of a in w iff it is the case that the agent *knows which object a is*, in the reading of [4,6,7]. The intuition behind this reading is that the satisfaction of the *de re* formula $\exists x K_i(x = a)$ requires that the interpretation of a is constant across i's epistemic alternatives. Hence, there is no uncertainty for i with respect to which object a is, and i is therefore able to identify a. Using a contingent identity system for objects, i.e. giving these a non-rigid interpretation as done in the semantics above, results in the invalidity

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[6] Though the agent may be able to tell them apart with respect to their color or position.

of both $(a = b) \rightarrow K_i(a = b)$ and $(a = b) \rightarrow \exists x K_i(x = b)$. Hence, agent i does *not* by default know object identities, and neither is the agent able to *identify* objects by default – as in the example above.

Word Lexicon. A suitable representation of the word lexicon is simpler to extract than for the semantic lexicon. This is due to the non-world relative interpretation of name constants $n \in LEX$, which so far has gone without comment. The interpretation function $\mathcal{I}$ of the name constants is defined constant in order ensure that the agent is *syntactically competent*. From the definition of $\mathcal{I}$, it follows that $(n_1 = n_2) \rightarrow K_i(n_1 = n_2)$ is valid on the defined class of models. This corresponds formally to the incontingent identity system used in [8]. The interpretation is that whenever the agent is presented by two name tokens of the same type of name, the agent knows that these are tokens of the same name type. The assumptions is adopted as the patients reviewed in [12] are able to recognize the words utilized.

Notice that identity statements such as $(n_1 = n_2)$ *do not* convey any information regarding the *meaning* of the names. Rather, they express identity of the two *signs*. Hence, the identity '*London* = *London*' is true, where as the identity '*London* = *Londres*' is false – as the two first occurrences of '*London*' are two tokens (e.g. $n_1, n_2 \in LEX$) of the same type (the type being $\dot{n} \in Nam$), whereas '*London*' and '*Londres*' are occurrences of two different name types, albeit with the same meaning.

Due to the simpler definition of $\mathcal{I}$ for name constants, we can define i's *name class for* n directly. Where $\dot{n} \in Nam$ and $n \in LEX$ this is the set $C_i^n(\dot{n}) = \{\dot{n}' : \mathcal{I}(n) = \dot{n}'\}$. The word lexicon of i is then the collection of such sets: $\mathsf{WL}_i = \{C_i^n(\dot{n}) : n \in LEX\}$. Each name class is a singleton equivalence class, and WL_i is a partition of Nam. Further, (1) (if suitably modified) holds also for name classes, and the construction of WL_i therefore fits nicely with the assumption of syntactic competence.

4.2 Interlude: Giving Words Meaning

In order to investigate the theory of Millian meaning, this theory must be embedded in the formal framework. This is simple due to the simplicity of the Millian theory: all it takes is for each name to be assigned a referent. To this effect we have in $\mathcal{L}$ included the function symbol μ. This is interpreted as a Millian meaning function. A function rather than a relation is used as only proper names are included in the agent's language, and for these to have unambiguous meanings, the function requirement is natural. Given it's defined arity, μ assigns a term from TER_{OBJ} to each term in TER_{LEX}. From the viewpoints of the agents, μ hence assigns an object to each name.

On the semantic level, we take $M, w \models_v (\mu(n) = a)$ to state that the meaning/referent of name n is the object a in the actual world w. The reference map is defined *world relatively*, i.e. the value $\mu(n)$ for $n \in LEX$, can change from world to world. This is the result of the world relative interpretation of μ given in the semantics above. Hence, *names are assigned values relative to epistemic alternatives*.

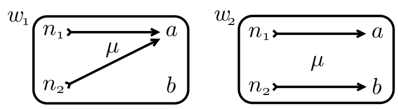

Fig. 2. The meaning function μ is defined world relatively, so meaning of a name may shift across epistemic alternatives

Two points need to be addressed here. One is the rigidity of names thus construed, and the other is knowledge of the reference of a name. With respect to the rigidity of the names in the present model, then they are indeed rigid, for they *do* refer to the same object in *every metaphysically possible world*. Note here, that it may be the case that the only metaphysically possible world included in the model is the actual world w, as all other elements of W are epistemic alternatives to w. All such epistemic alternatives may be metaphysically impossible under the assumption that names are *rigid designators*, cf. [11]. The epistemic alternatives can deviate from the actual world in any *logically possible way*[7]. This implies that *the meaning function is not by default known to the agents*. They may fail to know what object a given name refers to. On the other hand, the present modeling does not preclude such knowledge from being possible.

4.3 Competence Types

Inferential Competence. With respect to inferential competence, the present model is rather limited in the features expressible. This is a direct consequence of the simplifying assumptions. In particular, the limitation to proper names in the word lexicon limits the types of inferential competence to knowing relations between referring names, and thus precludes inferential knowledge regarding names and verbs. As an example, one cannot express that the agent knows the true sentence 'name is planet' as the word lexicon does not contain an entry for the verb 'is' nor for the predicate. As a result, it cannot be expressed, e.g., that an agent has the knowledge appropriate to retrieve a word from it's definition.

We are, however, able to express one feature of inferential competence important for the analysis of the Fregean argument, namely *knowledge of co-reference*:

$$K_i(\mu(n) = \mu(n')). \tag{2}$$

(2) states that i knows that n and n' mean the same, i.e., that the two names are Millian synonyms.

Based on (2), we may define that agent i is *generally inferentially competent with respect to n* by

$$M, w \models_v \forall \dot{x}((\mu(n) = \mu(\dot{x})) \rightarrow K_i(\mu(n) = \mu(\dot{x}))) \tag{3}$$

[7] Based on the present axiom system.

where $\dot{x} \in VAR_{LEX}$. If (3) is satisfied for all names n, agent i will have full 'encyclopedic' knowledge of the singular terms of her language. Alone, this will however be 'Chinese room style' knowledge, as it does not imply that the agent can apply any names, nor that the agent can name any objects.

Referential Competence. Regarding referential competence, recall that this compromises two distinct relations between names and objects, relating these through the semantic lexicon. The two relations are *application* and *naming*. An agent can *apply a name* if, when presented with a name, the agent can identify the appropriate referent. This ability can be expressed of the agent with respect to name n in w by

$$M, w \models_v \exists x K_i(\mu(n) = x) \tag{4}$$

i.e. there is an object which the agent can identify as being the referent of n. Given the assumption of syntactical competence, there is no uncertainty regarding which name is presented. Since the existential quantifier has scope over the knowledge operator, the interpretation of $\mu(n)$ is fixed across epistemic alternatives, and i thus knows which object n refers to.

To be able to *name an object*, the agent is required to be able to produce a correct name when presented with an object, say a. For this purpose, the *de re* formula $\exists \dot{x} K_i(\mu(\dot{x}) = a)$ is insufficient as $\mu(\dot{x})$ and a may simply co-vary across states. This means that i will be guessing *about which object is to be named*, and may therefore answer incorrectly. Since there may in this way be uncertainty regarding the presented object, naming must include a requirement that i can identify a, as well as know a name for a. This is captured by

$$M, w \models_v \exists x \exists \dot{x} K_i((x = a) \wedge (\mu(\dot{x}) = a)). \tag{5}$$

Here, the object quantification and first conjunction ensures that i can identify the presented object a and the second conjunct ensures that the name refers to a in all epistemic alternatives.

Dissociations. As mentioned, inferential competence and naming are dissociated. This is preserved in the model in that neither (2) nor (3) alone imply (5). Nor does (5) alone imply either of the two. The dissociation of application from naming is also preserved, as (4) does not alone entail (5). That application does not imply naming is illustrated in Figure 3.

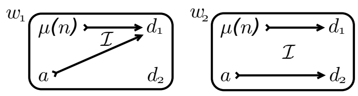

Fig. 3. Application and naming are not correlated. In actual world w_1, n refers to a and i can correctly apply n, but cannot name a using n: $w_1 \models_v (\mu(n) = a) \wedge \exists x K_i(\mu(n) = x)$, but $w_1 \models_v \neg \exists x \exists \dot{x} K_i((x = a) \wedge (\mu(\dot{x}) = a))$. Here, i cannot name a due to an ambiguous concept. a may be either of d_1 or d_2, and can therefore not be identified precisely enough to ensure a correct answer.

Whether application entails inferential competence, and whether naming entails application is not discussed in [12]. In the present model, however, these are modeled as dissociated in the sense that (4) does not entail, nor is entailed by, either (3) or (5). However, the modeled dissociations are *single instances* of the various abilities. Once more instances are regarded simultaneously, implicational relationships arise, as will be utilized in the analysis of the Fregean argument below.

A Weak Competence: Correlation. A further, albeit very weak, competence type can be found in the formal framework. This type emerges when the agent is able to correlate a name with an entry in the semantic lexicon, but where the latter is not an unambiguous concept. The ability is given by

$$M, w \models_v K_i \left(\mu\left(n\right) = a\right) \tag{6}$$

Here, the agent knows that the referent of the name n is co-extensional with i's concept a, but may be unable to identify which object a in fact is. We will refer to this ability as *correlation.*

4.4 Default Properties

To familiarize the reader with the class of models defined, a few properties are worth noting. First, we note that

$$K_i \forall x \exists \dot{x} (\mu(\dot{x}) = x) \tag{7}$$

stating that agent i knows of every object that it is named, is invalid on the set of models defined. This follows as we have not assumed μ surjective. Permuting the quantifiers results in the validity

$$K_i \forall \dot{x} \exists x (\mu(\dot{x}) = x) \tag{8}$$

capturing the idea that i knows that all names refer. In regard to [12], the validity of (8) is preferable, as non-denoting names where not used in the case-studies. Though i knows that all names refer, it is not assumed that the agent knows *what they refer to.* Hence,

$$\forall \dot{x} \exists x K_i (\mu(\dot{x}) = x) \tag{9}$$

is invalid on the set of models. This is natural as competence types are made as substantial assumptions.

5 Reviewing Frege's Puzzle

We now return to the argument presented in the introduction. Recall that where (a) is the identity statement 'Hesperus is Hesperus' and (b) is 'Hesperus is Phosphorus', the Fregean argument can be given the following structure:

(A) (a) and (b) mean the same.

(A→B) If (a) and (b) mean the same, then a semantically competent speaker would know that (a) and (b) mean the same.

(B→C) If a semantically competent speaker would know that (a) and (b) mean the same, then they are equally informative to the speaker.

(¬C) (a) and (b) differ in informativeness to the competent speaker.

∴ Contradiction.

The four premises are jointly inconsistent, and, as was mentioned, the typical textbook choice is to reject the premise (A).

Given the formal machinery introduced, it is now possible to evaluate this argument in a formal setting. The strategy used to evaluate the argument is to assume that the initial premise (A) is satisfied at actual word w in a model M, while also assuming that the agent is semantically competent, in each of three relevant ways. This results in three different versions of the argument: one for inferential competence, one for application and one for correlation. In the first two cases, the assumptions lead to satisfied versions of the premises (A→B) and (B→C), while making it clear why the 'intuitive' premise (¬C) should be rejected in these cases. In the final case, (¬C) cannot be rejected. However, due to the weak competence type used, the argument does not result in a contradiction, why it does not force the abandonment of Millianism.

Due to the restriction to a first-order language, it is not possible to properly represent the first premise, namely that the identity statements (a) and (b) mean the same. A proper representation would amount to something like

$$\mu(n \simeq n) \leftrightarrow \mu(n \simeq n') \tag{10}$$

where '$\simeq$' represents the word 'is' from the agent's language. Since this is not possible in $\mathcal{L}$, it is assumed that the first premise is natural language-equivalent with '(The meaning of 'Hesperus' is identical to the meaning of 'Hesperus') is equivalent with (The meaning of 'Hesperus' is identical to the meaning of 'Phosphorus')'. Under this assumption, the first premise may be represented by

$$(\mu(n) = \mu(n)) \leftrightarrow (\mu(n) = \mu(n')). \tag{11}$$

Since the left-hand identity is a validity, the first premise only amounts to the assumption that the actual world w in model M satisfies

$$(\mu(n) = \mu(n')). \tag{12}$$

The second premise is that (12) implies that any competent speaker knows that $(\mu(n) = \mu(n)) \leftrightarrow (\mu(n) = \mu(n'))$. The truth of this premise depends on the type of competence meant. The last three premises of the argument will be run through using inferential competence, application and correlation. The ability to name objects is not relevant for the present.

5.1 Inferential Competence

Casting the argument in terms of inferential competence, the second premise states that if n and n' mean the same, i.e. that (12) is satisfied, and agent i is inferential competent with respect to the two names, then agent i would know that n and n' mean the same. Recall that i is generally inferentially competent with respect to n iff

$$\forall \dot{x}((\mu(n) = \mu(\dot{x})) \rightarrow K_i(\mu(n) = \mu(\dot{x}))) \tag{13}$$

The antecedent of the second premise for inferential competence therefore becomes the conjunction of (12) and (13), and the consequent that (12) is known by i, i.e. that

$$K_i(\mu(n) = \mu(n')). \tag{14}$$

The full resulting second premise, that the conjunction of (12) and (13) imply (14), is a validity in relation to the semantics defined. By the initial assumption that (12) is satisfied, it therefore follows that

$$M, w \models_v K_i(\mu(n) = \mu(n')). \tag{15}$$

The third premise states that (14) implies that the two identity statements are equally informative to the agent. 'Equally informative' is here taken to mean that the two statements would eliminate the same worlds from agent i's model *if truthfully announced to the agent*, in the sense of [18]. As $(\mu(n) = \mu(n))$ is a validity, it eliminates no worlds, so the third premise can be reduced to (15) implying that

$$\neg\exists w' \sim_i w : M, w' \models_v \neg(\mu(n) = \mu(n')). \tag{16}$$

That no $\neg(\mu(n) = \mu(n'))$ world exists follows directly from (15) and the semantics of the K_i operator. Hence this premise holds true as well.

This is not the case with the last premise, namely that the identity statements should *not* be equally informative, i.e. that

$$\exists w' \sim_i w : M, w' \models_v \neg(\mu(n) = \mu(n')). \tag{17}$$

This premise is false as a consequence of the assumption of Millianism and agent i's inferential competence with respect to n and n'. However, that the agent will not be informed by the identity statement does not seem all that counter-intuitive *given the assumption of inferential competence.* The inferential competence of agent i is constituted by i's ability to find synonyms when prompted with names. As this is a knowledge-based ability, the knowledge that the identity statement is supposed to provide is already assumed to be possessed by the agent.

In short, if we stick with Millian meaning and assume agent i inferentially competent, i does not learn anything new by being told that the two names co-refer because this was assumed to be already known by i. This conclusion seems far from puzzling. In particular, it does not seem paradoxical enough (if at all) to warrant a rejection of the Millian view.

5.2 Referential Competence: Application

Turning to the argument utilizing application as the relevant competence type, the assumption that agent i is semantically competent with respect to n and n' results in the assumption that i can apply both names. Recall that i can apply the name n at w iff $M, w \models_v \exists x K_i(\mu(n) = x)$, i.e., there is an object which i can identify as being the referent of n.

With the assumption that i can apply both n and n' in the antecedent, the second premise is captured by

$$(\mu(n) = \mu(n')) \wedge \exists x K_i(\mu(n) = x) \wedge \exists y K_i(\mu(n') = y) \rightarrow K_i(\mu(n) = \mu(n')). \quad (18)$$

The formula (18) is valid on the class of models defined, and is therefore also satisfied at w in M. As the antecedent is assumed satisfied, the consequent (B) from the second premise will, as in the previous case, follow. I.e., it is concluded that $M, w \models_v K_i(\mu(n) = \mu(n'))$.

The third premise can be formulated as it was in the previous case, and given that $M, w \models_v K_i(\mu(n) = \mu(n'))$ holds, it will again follow that the agent will not be informed by the identity statement, i.e. that the statement eliminates no worlds: $\neg\exists w' \sim_i w : M, w' \models_v \neg(\mu(n) = \mu(n'))$.

Hence, if one assumes that n and n' co-refer, and that the agent is able to apply both of these names, then one is forced to reject the the premise ($\neg$C), that the agent will be informed.

Yet, one may still feel that this argument does not provide ample reason to give up the intuitions behind ($\neg$C). In particular, one may object to the validity of (18). One argument can be based on exactly on the case involving Hesperus and Phosphorus. One could easily envision an agent able to identify Venus as the referent of 'Hesperus' in the evening and as the referent of 'Phosphorus' in the morning, while still being unaware that these two names co-refer. Exactly this objection is raised in [15], where it is argued that the objection contains an appeal to *contexts* not captured in the present models. However, if contexts are added to the formal setting and suitable, context-dependent competence types are defined, the objection can be avoided, cf. [15, ch. 7].

In the present work, the model is only constructed to deal with the mono-context case, fitting, e.g., the interview scenarios used when testing the linguistic abilities of various brain injured subjects. Within the same context, the validity of (18) is easy to justify. Assume a person in the presence of a number of items is given a name of one of them, and successfully applies the name, i.e., successfully identifies the object to which the name refers, using his knowledge-based identification skills regarding that name and object. The task performed to identify the proper object could for example be to place a note with the name on the object. Assume the same task is repeated with the same successful outcome, but a different name referring to the same object. Given suitable assumptions regarding short-term memory and minimal deductive abilities, the agent should now know that the two names refer to the same object. In fact, this should not be much harder for the agent than to realize that the two notes just placed are stuck on the same object.

To summarize, if we stick with Millian meaning and assume that the agent can identify both referents, then she does not learn anything new by being told that the two names co-refer. Further, that the agent is not informed is a natural consequence of the assumptions made regarding the agent's semantic competence. Therefore, the intuitively correct premise (¬C) should be rejected.

5.3 Weak Competence: Correlation

For the third version of the argument, we turn to a weaker notion of semantic competence, namely correlation. Running through the argument using this weaker ability, the second premise becomes

$$K_i(\mu(n) = a) \wedge K_i(\mu(n') = b) \rightarrow K_i(\mu(n) = \mu(n')) \tag{19}$$

This formula is satisfiable, but not valid in the defined class of models. This means that $M, w \models_v K_i(\mu(n) = \mu(n'))$ will be true or false depending on the specific model. In case the consequent of (19) is satisfied, the agent will have knowledge of co-reference, and it will, like above, not be surprising that he is not informed by the identity statement.

In case the consequent fails, a new situation arises. In particular, this will imply that (16) likewise fails to be the true. From this it follows that the premise (¬C)

$$\exists w' \sim_i w : M, w' \models_v \neg(\mu(n) = \mu(n'))$$

now holds, as opposed to the above cases. This in turn means that the agent *will* be informed by the identity statement. If a truthful announcement of the identity statement is made to the agent, any w' as specified in (17) can be eliminated, and the agent will thereby gain information.

By the truthful announcement, the agent is informed on both an *inferential* and a *conceptual* level. First, the agent will after the announcement have knowledge of co-reference with respect to the two names. Secondly, where the agent before had two distinct concepts, the agent's concepts of a and b will after the announcement have merged.

However, given the weaker notion of competence, that the agent is informed does not conflict with the assumption of Millian meaning of proper names. To see this, notice that the two premises (A→B) and (B→C) from the argument above are false when assuming this weaker form of semantic competence. As a result, the problematic contradiction no longer follows, and Millianism and the requirement that the agent should be informed by the identity statement can therefore be unified.

To sum up, neither of the three arguments provide a strong basis for rejecting Millianism. If inferential competence is assumed, then the knowledge supposedly provided by the identity statement is directly assumed already. If the agent is supposed to be able to apply both names, it should also be able to deduce that the names denote the same object, why the identity statement will not be informative. Finally, if one assumes that the agent is weakly competent enough to be informed, the contradiction problematic for the Millian cannot be derived.

6 Conclusions and Further Perspectives

The theory of lexical competence from [12] has been modeled, and the key elements of the structure preserved. In the model, the three types of lexical competence proposed in [12] were identified along with a fourth which had not been considered in the original text. When regarding Frege's Puzzle in a formal setting using the relevant types of competence, it was seen that each argument was far from all being as decisive against Millianism as has been the mainstream assumption in 20th century philosophy of language.

One issue for further research would be to investigate whether light can be thrown on other puzzles from the philosophy of language by focusing on the epistemic states of the language user, rather than on semantic theories of the language. It would further be interesting to investigate the model in more details, and compare this to newer literature from cognitive neuropsychology. One could suspect that a more fine grained view of semantic competence was required. One obvious way to gain such would be to use weaker operators like those presented in [1]. Using weaker modalities to model semantic competence could possibly result in levels where individual concepts contain no existing objects. This could possibly shed light on problems of reference to non-existing objects. Finally, a logic of language is not much fun in the mono-agent case. In order to investigate how lacking semantic competence influences communication and action in multi-agent settings, it would be interesting to move to a dynamic framework.

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